CS Basics
Bases 2 et 16

Fall Term 2025/2026

Emmanuel Benoist | BFH-TI

Bases

- **Last Week**
- **What is a base?**
- **Hexadecimal**
  Conversions
- **Binary**
  Hex as shorthand for binary

Last Week

Architecture

A program is just data

- Instructions are data
- They are bits, and are represented as numbers (in hex for instance)

Memory and CPU are linked throw a Bus

- It links also all periferals
- CPU writes on it and periferals read what is for them

What is a base?

We count in base 10

Signs used for counting

- $1, 2, 3, 4, 5, 6, 7, 8, 9$, and 0
- a number is a list of signs 123 means $1 \times 100 + 2 \times 10 + 3$.

Other bases were used over the time

- Base 12 (for hours for instance)
- Base 60 (for minutes for instance) 50 minutes and 33 seconds is a time counted in bases 60,
  It makes $50 \times 60 + 33 = 3033$ seconds
  btw: $60 = 5 \times 12$

In the 70's other bases were used to teach counting

- "Modern mathematics"

Essence of a number base

Romans used a system where letters represented values

- MMXIV means 2014
- MCMXC means 1990
- Le positions of the letters are not bounded to a column, but rather to their neighbors (for adding or substracting)

We use only columnar systems: the position of a number means the value

- in every base, the number 10 represents the base
- Number in column number 0 is multiplied by $base^0 = 1$
- Number in column number 1 is multiplied by $base^1 = 10_{base}$
- Number in column number 2 is multiplied by $base^2 = 100_{base}$
- Number in column number 3 is multiplied by $base^3 = 1000_{base}$

Hexadecimal

Hexadecimal

Hexadecimal = base 16

- Is the real base for programmers

Digits

- 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, ...
- A (10), B (11), C (12), D (13), E (14), F (15)
- 16 is the base and therefore written 10 in Hexadecimal

Counting

- 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 1A, 1B, 1C, 1D, 1E, 1F, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 2A, 2B, 2C, 2D, 2E, 2F,30

Notation

- In the remainder of this course, we will denote Hexadecimal numbers with a prefix 0x.
- So 1 becomes 0x1 or 2F becomes 0x2F

Hexadecimal table

$0x0$	zero	0
$0x1$	one	1
$0x2$	two	2
$0x3$	three	3
$0x4$	four	4
$0x5$	five	5
$0x6$	six	6
$0x7$	seven	7
$0x8$	eight	8
$0x9$	nine	9
$0xA$	A	10
$0xB$	B	11
$0xC$	C	12
$0xD$	D	13
$0xE$	E	14
$0xF$	F	15

$0x10$	One -oh hexadecimal	16
$0x11$	One -one hexadecimal	17
$0x12$	One -two hex.	18
$0x13$	One -three hex.	19
$0x14$	One -four hex.	20
$0x15$	One -five hex.	21
$0x16$	One -six hex.	22
$0x17$	One -seven hex.	23
$0x18$	One -eight hex.	24
$0x19$	One -nine hex.	25
$0x1A$	One -A hex.	26
$0x1B$	One -B hex.	27
$0x1C$	One -C hex.	28
$0x1D$	One -D hex.	29
$0x1E$	One -E hex.	30
$0x1F$	One -F hex.	31
$0x20$	Two -oh hex.	32

Table of powers of 16

Hexadecimal uses powers of 16

$0x1$	16^0	1
$0x10$	16^1	16
$0x100$	16^2	256
$0x1000$	16^3	4096
$0x10000$	16^4	65536
$0x100000$	16^5	1048576
$0x1000000$	16^6	16777216

Anatomy of a number

Let us evaluate the number $0x3C0A9$

$$
\begin{array}{cccccc}
 & & & & & 0x9 \\
 & & & & 0xA & 0 \\
 & & & 0x0 & 0 & 0 \\
 & & 0xC & 0 & 0 & 0 \\
+ & 0x3 & 0 & 0 & 0 & 0 \\
\hline
 & 0x3 & C & 0 & A & 9
\end{array}
$$

$$3 \times 65536 + 12 \times 4096 + 0 \times 256 + 10 \times 16 + 9 \times 1\ 196608 + 49152 + 0 + 160 + 9 = 245929$$

From Hex to Decimal

Method

- Compute the value of each column and add the results

Decimal value of $0x7A2$

- add 2 (for the 2 in column 0)
- add 10×16 (for the A in column 1)
- add 7×256 (for the 7 in column 2)

Other example

Value of $0xC6F0DB$

- $B \times 1 = 11$
- $D \times 16 = 13 \times 16 = 208$
- $0 \times 256 = 0$
- $F \times 4096 = 15 \times 4096 = 61\,440$
- $6 \times 65\,536 = 393\,216$
- $C \times 1\,048\,576 = 12 \times 1\,048\,576 = 12\,582\,912$

Total = 13 037 787

From Decimal to Hex

We want to write 450 in hex.

- Find the largest hex column value that is contained at least once in 450
  4096 is to large, and 256 is perfect.
- find which number of 256 goes into 450 (remember 5rd Grade division)
- $449/256 = 1.7539$ so 1 is the lefmost hex digit
- Let us substract 1×256 from 450, we obtain 194
- The next power of 16 is 16 itself, how many times 16 goes into 194
- $194/16 = 12.065$, so C is the next hex digit
- $194 - 12 * 16 = 2$ the remainder is one and the next value of 16 (i.e. 16^0) is 1, so the next hex digit is 2

The hex value of 450 is $0x1C2$

Another example

What is the hex value of $988\ 664$

- The largest power of 16 contained in the number is $65\ 536$
- $65\ 536$ goes 15 times in $988\ 664$, so the left most hex digit is **F**.
- The remainder is $988\ 664 - 65\ 536 \times 15 = 5624$
- The next power of 16 is 4096, which goes only once in 5624. So the next hex digit is **1**.
- The remainder is $5624 - 4096 = 1528$
- The next power of 16 is 256, 256 goes 5 times into 1528
- The next hex digit is **5**
- The remainder is $1528 - 5 * 256 = 248$
- The next of 16 is 16 itself, 16 goes 15 times into 148
- The next hex digit is **F**
- The remainder is $148 - 16 \times 15 = 8$
- The last hex digit is **8**

The hex value is $0xF15F8$

Columns and carries

Method for adding hex numbers

Add each column starting from the right and carry into the next column anytime the result exceeds $0FH$

$$
\begin{array}{rcccccc}
 & 1 & & & & 1 & \\
 & 0x2 & F & 3 & 1 & A & D \\
+ & 0x9 & 6 & B & A & 0 & 7 \\
\hline
 & 0xC & 5 & E & B & B & 4
\end{array}
$$

- The most you can carry is `1`

Substraction and borrows

We have to mentally reverse

- if $E + 6 = 0x14$ then $0x14 - 6 = E$

We have to substract column by column

- Start from right

$$
\begin{array}{rcccc}
 & 0xF & 7 & 6 & C \\
- & 0xA & 0 & 5 & B \\
\hline
 & 0x5 & 7 & 1 & 1
\end{array}
$$

Borrows

Need for borrows if a value to substract is larger than the one we substract from.

- $9 - A = ???$

$$\begin{array}{rrr} & 0x9 & 2 \\ - & 0x4 & F \\ \hline & ? & ? \end{array}$$

- We need to add `0x10` (i.e. 16_{10} to the number for the substraction to be possible.

$$\begin{array}{rrr} & 0x9 & 2 \\ - & 0x4_1 & F \\ \hline & 0x4 & 3 \end{array}$$

Borrows accross Multiple Columns

We may have to transfer the borrow accross more than one column

$$\begin{array}{rcccc} & 0xF & 0 & 0 & 0 \\ - & 0x3 & B & 6 & C \\ \hline & 0x? & ? & ? & ? \end{array}$$

$$\begin{array}{rcccc} & 0xF & 0 & 0 & 0 \\ - & 0x3 & B & 6_1 & C \\ \hline & 0x? & ? & ? & 4 \end{array}$$

$$\begin{array}{rcccc} & 0xF & 0 & 0 & 0 \\ - & 0x3 & B_1 & 6_1 & C \\ \hline & 0x? & ? & 9 & 4 \end{array}$$

Borrows accross Multiple Columns

$$
\begin{array}{rcccc}
 & 0xF & 0 & 0 & 0 \\
- & 0x3_1 & B_1 & 6_1 & C \\
\hline
 & 0x? & 4 & 9 & 4
\end{array}
$$

$$
\begin{array}{rcccc}
 & 0xF & 0 & 0 & 0 \\
- & 0x3_1 & B_1 & 6_1 & C \\
\hline
 & 0xB & 4 & 9 & 4
\end{array}
$$

Binary

Binary

There are only two digits (0 and 1) in the base

Each column has a value two times the column to its right

0

1

10

11

100

101

110

111

1000

1001

1010

Powers of 2

Binary	Power of 2	decimal
1	2^0	1
10	2^1	2
100	2^2	4
1000	2^3	8
10000	2^4	16
100000	2^5	32
1000000	2^6	64
10000000	2^7	128
100000000	2^8	256
1000000000	2^9	512
10000000000	2^{10}	1024
100000000000	2^{11}	2048
1000000000000	2^{12}	4096
10000000000000	2^{13}	8192

Notation

Values in binary should be noted with the prefix ob
$0b110$ means 6
whereas
$0x110$ means 272
and 110 means 110

Notations in scientific books use subscript
110_2 means 6_{10}
whereas
110_{16} means 272_{10}
and 110_{10} means 110_{10}

But it is not usable inside source files or simple text editors.

Why are computer binary?

Other machines have been tested with base 3

- 1840 Thomas Fowler built a ternary calculating machine from wood
- 1958 Nikolay Brusentsov (USSR) built the *Setun* computer
- in 1973 he built an enhanced version called *Setun-70*
- In the USA, a computer was built in 1973 *Ternac*

Because lights are either on or off

- In an electrical device : voltage is present or not
- It means 1 or 0

Hex as shorthand for binary

218 **is expressed in binary:** $0b11011010$

expressed in hex: $0xDA$

- $0xA$ (or $0x0A$ in assembler) represents the number 10
- Conversion in Binary: $0b1010$
- They are the last four digits of 218 in binary
- $0xD$ is also $0b1101$

218		decimal
1101	1010	binary
D	A	hex

Hex as shorthand for binary

If we have a 32 binary number
$0b11110000000000001111101001101110$

We can split it into groups of 4 digits
$1111\ 0000\ 0000\ 0000\ 1111\ 1010\ 0110\ 1110$

Each group of 4 is represented by one Hex value

$$\begin{array}{cccccccc} 1111 & 0000 & 0000 & 0000 & 1111 & 1010 & 0110 & 1110 \\ F & 0 & 0 & 0 & F & A & 6 & E \end{array}$$

The hex equivalent is $0xF000FA6E$

Conclusion

Computers work only in binary

- Notations in Binary are too long
- We use hex to represent binary values

You should be familiar with hex notation

- It is the center of low level programming.
- One solution: do the exercises!

Source

Book: Assembly Language Step by Step (3rd edition) by Jeff Duntemann