

CS Basics

Bases 2 et 16

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Last Week

Bases

- ▶ Last Week
- ▶ What is a base?
- ▶ Hexadecimal Conversions
- ▶ Binary
Hex as shorthand for binary

Architecture

A program is just data

- Instructions are data
- They are bits, and are represented as numbers (in hex for instance)

Memory and CPU are linked throw a Bus

- It links also all peripherals
- CPU writes on it and peripherals read what is for them

What is a base?

Essence of a number base

Romans used a system where letters represented values

- MMXIV means 2014
- MCMXC means 1990
- The positions of the letters are not bounded to a column, but rather to their neighbors (for adding or subtracting)

We use only columnar systems: the position of a number means the value

- in every base, the number 10 represents the base
- Number in column number 0 is multiplied by $base^0 = 1$
- Number in column number 1 is multiplied by $base^1 = 10_{base}$
- Number in column number 2 is multiplied by $base^2 = 100_{base}$
- Number in column number 3 is multiplied by $base^3 = 1000_{base}$

We count in base 10

Signs used for counting

- 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0
- a number is a list of signs 123 means $1 \times 100 + 2 \times 10 + 3$.

Other bases were used over the time

- Base 12 (for hours for instance)
- Base 60 (for minutes for instance) 50 minutes and 33 seconds is a time counted in bases 60. It makes $50 \times 60 + 33 = 3033$ seconds
btw: $60 = 5 \times 12$

In the 70's other bases were used to teach counting

- "Modern mathematics"

Hexadecimal

From Hex to Decimal

Method

- Compute the value of each column and add the results

Decimal value of $0x7A2$

- add 2 (for the 2 in column 0)
- add 10×16 (for the A in column 1)
- add 7×256 (for the 7 in column 2)

From Decimal to Hex

We want to write 450 in hex.

- Find the largest hex column value that is contained at least once in 450
4096 is too large, and 256 is perfect.
- find which number of 256 goes into 450 (remember 5th Grade division)
- $449/256 = 1.7539$ so 1 is the leftmost hex digit
- Let us subtract 1×256 from 450, we obtain 194
- The next power of 16 is 16 itself, how many times 16 goes into 194
- $194/16 = 12.065$, so C is the next hex digit
- $194 - 12 * 16 = 2$ the remainder is one and the next value of 16 (i.e. 16^0) is 1, so the next hex digit is 2

The hex value of 450 is $0x1C2$

Other example

Value of $0xC6F0DB$

- $B \times 1 = 11$
- $D \times 16 = 13 \times 16 = 208$
- $0 \times 256 = 0$
- $F \times 4096 = 15 \times 4096 = 61\,440$
- $6 \times 65\,536 = 393\,216$
- $C \times 1\,048\,576 = 12 \times 1\,048\,576 = 12\,582\,912$

Total = **13 037 787**

Another example

What is the hex value of 988 664

- The largest power of 16 contained in the number is 65 536
- 65 536 goes 15 times in 988 664, so the left most hex digit is F.
- The remainder is $988\,664 - 65\,536 \times 15 = 5624$
- The next power of 16 is 4096, which goes only once in 5624. So the next hex digit is 1.
- The remainder is $5624 - 4096 = 1528$
- The next power of 16 is 256, 256 goes 5 times into 1528
- The next hex digit is 5
- The remainder is $1528 - 5 * 256 = 248$
- The next of 16 is 16 itself, 16 goes 15 times into 148
- The next hex digit is F
- The remainder is $148 - 16 \times 15 = 8$
- The last hex digit is 8

The hex value is $0xF15F8$

Borrows across Multiple Columns

$$\begin{array}{r}
 0xF \quad 0 \quad 0 \quad 0 \\
 - 0x3_1 \quad B_1 \quad 6_1 \quad C \\
 \hline
 0x? \quad 4 \quad 9 \quad 4 \\
 \\
 0xF \quad 0 \quad 0 \quad 0 \\
 - 0x3_1 \quad B_1 \quad 6_1 \quad C \\
 \hline
 0xB \quad 4 \quad 9 \quad 4
 \end{array}$$

Binary

There are only two digits (0 and 1) in the base

Each column has a value two times the column to its right

0
1
10
11
100
101
110
111
1000
1001
1010

Binary

Powers of 2

| Binary | Power of 2 | decimal |
|----------------|------------|---------|
| 1 | 2^0 | 1 |
| 10 | 2^1 | 2 |
| 100 | 2^2 | 4 |
| 1000 | 2^3 | 8 |
| 10000 | 2^4 | 16 |
| 100000 | 2^5 | 32 |
| 1000000 | 2^6 | 64 |
| 10000000 | 2^7 | 128 |
| 100000000 | 2^8 | 256 |
| 1000000000 | 2^9 | 512 |
| 10000000000 | 2^{10} | 1024 |
| 100000000000 | 2^{11} | 2048 |
| 1000000000000 | 2^{12} | 4096 |
| 10000000000000 | 2^{13} | 8192 |

Notation

Values in binary should be noted with the prefix ob

0b110 means 6

whereas

0x110 means 272

and 110 means 110

Notations in scientific books use subscript

110₂ means 6₁₀

whereas

110₁₆ means 272₁₀

and 110₁₀ means 110₁₀

But it is not usable inside source files or simple text editors.

Hex as shorthand for binary

218 is expressed in binary: 0b11011010

expressed in hex: 0xDA

- 0xA (or 0x0A in assembler) represents the number 10
- Conversion in Binary: 0b1010
- They are the last four digits of 218 in binary
- 0xD is also 0b1101

| | |
|-----------|---------|
| 218 | decimal |
| 1101 1010 | binary |
| D A | hex |

Why are computer binary?

Other machines have been tested with base 3

- 1840 Thomas Fowler built a ternary calculating machine from wood
- 1958 Nikolay Brusentsov (USSR) built the *Setun* computer
- in 1973 he built an enhanced version called *Setun-70*
- In the USA, a computer was built in 1973 *Ternac*

Because lights are either on or off

- In an electrical device : voltage is present or not
- It means 1 or 0

Hex as shorthand for binary

If we have a 32 binary number

0b11110000000000001111101001101110

We can split it into groups of 4 digits

1111 0000 0000 0000 1111 1010 0110 1110

Each group of 4 is represented by one Hex value

| | | | | | | | |
|------|------|------|------|------|------|------|------|
| 1111 | 0000 | 0000 | 0000 | 1111 | 1010 | 0110 | 1110 |
| F | 0 | 0 | 0 | F | A | 6 | E |

The hex equivalent is 0xF000FA6E

Conclusion

Computers work only in binary

- Notations in Binary are too long
- We use hex to represent binary values

You should be familiar with hex notation

- It is the center of low level programming.
- One solution: do the exercises!

Source

Book: Assembly Language Step by Step (3rd edition) by Jeff Duntemann